Chapter 6

Meeting Standards and Standings

In This Chapter

- How to standardize scores
- Making comparisons
- Ranks in files
- Rolling in the percentiles

In my left hand I hold 15 Argentine pesos. In my right, I hold 100 Chilean pesos. Which is worth more? Both currencies are called *pesos*, right? So shouldn't the 100 be greater than the 15? Not necessarily. "Peso" is just word magic — a coincidence of names. Each one comes out of a different country, and each country has its own economy. To compare the two amounts of money, you have to convert each currency into a standard unit. The most intuitive standard for us is our own currency. How much is each amount worth in dollars and cents? As I write this, 15 Argentine pesos are worth more than $4. One hundred Chilean pesos are worth about 15 cents.

In this chapter, I show you how to use statistics to create standard units. Standard units show you where a score stands in relation to other scores in a group, and I show you additional ways to determine a score's standing within a group.

Catching Some Zs

As the previous paragraph shows, a number in isolation doesn't really tell a story. In order to fully understand what a number means, you have to consider the process that produced it. In order to compare one number to another, they both have to be on the same scale.

In some cases, like currency conversion, it's easy to figure out a standard. In others, like temperature conversion or conversion into the metric system, a formula guides you.

When it's not all laid out for you, you can use the mean and the standard deviation to standardize scores that come from different processes. The idea is to take a set of scores and use its mean as a zero-point and its standard deviation as a unit of measure. Then you compare the deviation of each score from the mean to the standard deviation. You're asking "how big is a particular deviation relative to (something like) an average of all the deviations?"

To do this, you divide the score's deviation by the standard deviation. In effect, you transform the score into another kind of score. The transformed score is called a *standard score*, or a *z-score*.

The formula for this is

$$z = \frac{X - \bar{X}}{s}$$

if you're dealing with a sample, and

$$z = \frac{X - \mu}{\sigma}$$

if you're dealing with a population. In either case, x represents the score you're transforming into a z-score.

Characteristics of z-scores

A z-score can be positive, negative, or zero. A negative z-score represents a score that's less than the mean and a positive z-score represents a score that's greater than the mean. When the score is equal to the mean, its z-score is zero.

When you calculate the z-score for every score in the set, the mean of the z-scores is 0, and the standard deviation of the z-scores is 1.

After you do this for several sets of scores, you can legitimately compare a score from one set to a score from another. If the two sets have different means and different standard deviations, comparing without standardizing is like comparing apples with kumquats.

In the examples that follow, I show how to use z-scores to make comparisons.

Bonds versus The Bambino

Here's an important question that often comes up in the context of serious metaphysical discussions: Who is the greatest home run hitter of all time, Barry Bonds or Babe Ruth? Although this is a difficult question to answer, one way to get your hands around it is to look at each player's best season and compare the two. Bonds hit 73 home runs in 2001, and Ruth hit 60 in 1927. On the surface, Bonds appears to be the more productive hitter.

The year 1927 was very different from 2001, however. Baseball (and everything else) went through huge changes in the intervening years, and player statistics reflect those changes. A home run was harder to hit in the 20s than in the 00s. Still, 73 versus 60? Hmmm . . .

Standard scores can help us decide whose best season was better. To standardize, I took the top 50 home run hitters of 1927 and the top 50 from 2001. I calculated the mean and standard deviation of each group, and then turned Ruth's 60 and Bonds's 73 into z-scores.

The average from 1927 is 12.68 homers with a standard deviation of 10.49. The average from 2001 is 37.02 homers with a standard deviation of 9.64. Although the means differ greatly, the standard deviations are pretty close.

And the z-scores? Ruth's is

$$z = \frac{60 - 12.68}{10.49} = 4.51$$

Bonds's is

$$z = \frac{73 - 37.02}{9.64} = 3.73$$

The clear winner in the z-score best-season home run derby is Babe Ruth. Period.

Just to show you how times have changed, Lou Gehrig hit 47 home runs in 1927 (finishing second to Ruth) for a z-score of 3.27. In 2001, 47 home runs amounted to a z-score of 1.04.

Exam scores

Getting away from sports debates, one practical application of z-scores is the assignment of grades to exam scores. Based on percentage scoring, instructors traditionally evaluate a score of 90 points or higher (out of 100) as an A, 80–89 points as a B, 70–79 points as a C, 60–69 points as a D, and less than 60 points as an F. Then they average scores from several exams together to assign a course grade.

Is that fair? Just as a peso from Argentina is worth more than a peso from Chile, and a home run was harder to hit in 1927 than in 2001, is a point on one exam worth the same as a "point" on another? Like peso, isn't that just word magic?

Indeed it is. A point on a difficult exam is, by definition, harder to come by than a point on an easy exam. Because points might not mean the same thing from one exam to another, the fairest thing to do is convert scores from each exam into z-scores before averaging them. That way, you're averaging numbers on a level playing field.

In the courses I teach, I do just that. I often find that a lower numerical score on one exam results in a higher z-score than a higher numerical score from another exam. For example, on an exam where the mean is 65 and the standard deviation is 12, a score of 71 results in a z-score of .5. On another exam, with a mean of 69 and a standard deviation of 14, a score of 75 is equivalent to a z-score of .429. (Yes, it's like Ruth's 60 home runs versus Bonds's 73.) Moral of the story: Numbers in isolation tell you very little. You have to understand the process that produces them.

STANDARDIZE

Excel's STANDARDIZE worksheet function calculates z-scores. Figure 6-1 shows a set of exam scores along with their mean and standard deviation. I used AVERAGE and STDEVP to calculate the statistics. The Function Arguments dialog box for STANDARDIZE is also in the figure.

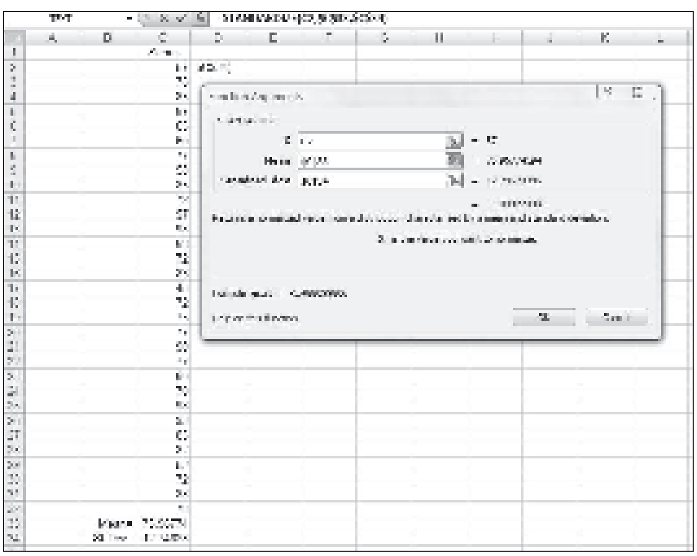


Figure 6-1: Exam scores and the Function Arguments dialog box for STANDARDIZE.

Caching some z's

Because negative z-scores might have connotations that are, well, negative, educators sometimes change the z-score when they evaluate students. In effect, they're hiding the z-score, but the concept is the same — standardization with the standard deviation as the unit of measure.

One popular transformation is called the T-score. The T-score eliminates negative scores because a set of T-scores has a mean of 50 and a standard deviation of 10. The idea is to give an exam, grade all the tests, and calculate the mean and standard deviation. Next, turn each score into a z-score. Then follow this formula:

$$T = (z)(10) + 50$$

People who use the T-score often like to round to the nearest whole number.

SAT scores are another transformation of the z-score. (Some refer to the SAT as a C-score.) The SAT has a mean of 500 and a standard deviation of 100. After the exams are graded, and their mean and standard deviation calculated, each exam score becomes a z-score in the usual way. This formula converts the z-score into a SAT score:

$$SAT = (z)(100) + 500$$

Rounding to the nearest whole number is part of the procedure here, too.

The IQ score is still another transformed z. Its mean is 100 and (in the Stanford-Binet version) its standard deviation is 16. What's the procedure for computing an IQ score? You guessed it. In a group of IQ scores, calculate the mean and standard deviation, and then calculate the z-score. Then it's

$$IQ = (z)(16) + 100$$

As with the other two, IQ scores are rounded to the nearest whole number.

Here are the steps:

1. **Enter the data into an array and select a cell.**

   The data are in C2:C32. I selected D2 to hold the z-score for the score in C2. Ultimately, I'll autofill column D and line up all the z-scores next to the corresponding exam scores.

2. **From the Statistical Functions menu, select STANDARDIZE to open the Function Arguments dialog box for STANDARDIZE.**

3. **In the Function Arguments dialog box, enter the appropriate values for the arguments.**

   First, I entered the cell that holds the first exam score into the X box. In this example, that's D2.

   In the Mean box, I entered the cell that holds the mean — C33 for this example. It has to be in absolute reference format, so the entry is $C$33.

You can type it that way, or you select C33 and then highlight the Mean box and press the F4 key.

In the Standard_dev box, I entered the cell that holds the standard deviation. The appropriate cell in this example is C34. This also has to be in absolute reference format, so the entry is $C$34.

4. **Click OK to close the Function Arguments dialog box and put the z-score for the first exam score into the selected cell.**

To finish up, I positioned the cursor on the selected cell's autofill handle, hold the left mouse button down, and drag the cursor to autofill the remaining z-scores.

Figure 6-2 shows the autofilled array of z-scores.

D2 =STANDARDIZE(C2,$C$33,$C$34)

	A	B	C	D	E	F	G
1			Score				
2			57	-1.4881			
3			70	0.155439			
4			85	0.708619			
5			87	0.865527			
6			66	-0.85047			
7			90	1.25274			
8			77	0.080085			
9			90	1.40956			
10			85	0.708619			
11			72	0.31129			
12			57	-1.4881			
13			90	1.728624			
14			88	0.943082			
15			72	0.31129			
16			83	0.55171			
17			45	2.42956			
18			72	-0.31129			
19			75	0.07592			
20			77	0.080085			
21			90	1.10059			
22			77	0.080085			
23			80	0.316040			
24			70	-0.46819			
25			90	1.728624			
26			93	1.386253			
27			83	0.55171			
28			82	0.473256			
29			40	1.02192			
30			72	-0.31129			
31			80	0.943982			
32			78	0.155439			
33		Mean=	75.95774				
34		St Dev=	12.74628				
35							

Figure 6-2: The autofilled array of z-scores.

Where Do You Stand?

Standard scores are designed to show you how a score stands in relation to other scores in the same group. To do this, they use the standard deviation as a unit of measure.

If you don't want to use the standard deviation, you can show a score's relative standing in a simpler way. You can determine the score's rank within the group: The highest score has a rank of 1, the second highest has a rank of 2, and so on.

RANK

With Excel's RANK function you can quickly determine the ranks of all the scores in a group. Figure 6-3 shows the Function Arguments dialog box for RANK along with a group of scores. I've also set up a column for the ranks.

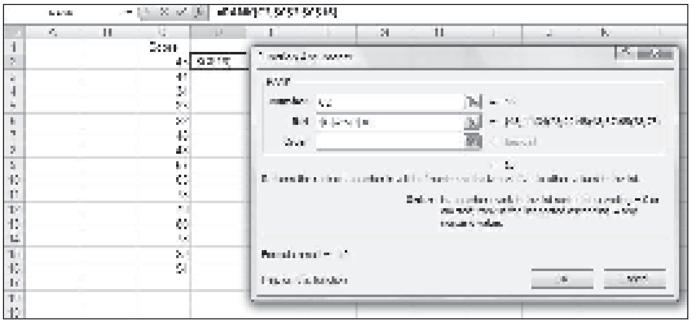


Figure 6-3: Working with RANK.

Here are the steps for using RANK:

1. **Enter the data into an array and select a cell.**

   For this example, I entered the scores into cells C2 through C16, and selected cell D2.

2. **From the Statistical Functions menu select RANK to open the Function Arguments dialog box for RANK.**

3. **In the Function Arguments dialog box, type the appropriate values for the arguments.**

   In the Number box, I entered the cell that holds the score whose rank I want to insert into the selected cell. For this example, that's C2.

   In the Ref box, I entered the array that contains the scores. I enter C2:C16 into the Ref box.

   This part is important. After I insert RANK into D2 I'm going to drag the cursor through column D and autofill the ranks of the remaining scores. To set up for this, I have to let Excel know I want C2 through C16 to be the array for every score, not just the first one.

That means the array in the Ref box has to look like this: $C$2:$C$16. I can either add the $-signs manually, or highlight the Ref box and then press the F4 key.

In the Order box, I indicate the order for sorting the scores. To rank the scores in descending order, I can either leave the Order box alone or type 0 (zero) into that box. To rank the scores in ascending order, I type a non-zero value into the Order box. I left this box alone.

4. **Click OK to put the rank into the selected cell.**

I then position the cursor on the selected cell's autofill handle, hold the left mouse button down, and drag the cursor to autofill the ranks of the remaining scores. (See Figure 6-4.)

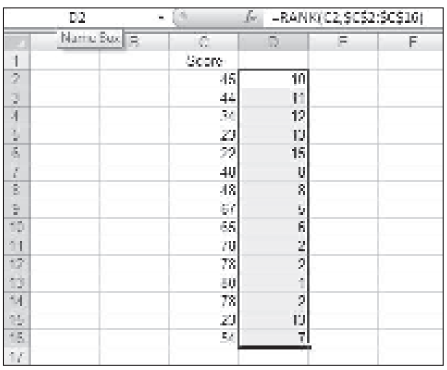


Figure 6-4: The auto-filled ranks.

LARGE and SMALL

You can turn the ranking process inside out by supplying a rank and asking which score has that rank. The worksheet functions LARGE and SMALL handle this from either end. They tell you the fifth largest score or the third smallest score, or any other rank you're interested in.

Figure 6-5 shows the Function Arguments dialog box for LARGE. In the Array box you enter the array of cells that holds the group of scores. In the K box you enter the position whose value you want to find. To find the seventh largest score in the array, for example, type **7** into the K box.

SMALL does the same thing, except it finds score positions from the lower end of the group. The Function Arguments dialog box for SMALL also has an Array box and a K box. Entering 7 in this K box returns the seventh lowest score in the array.

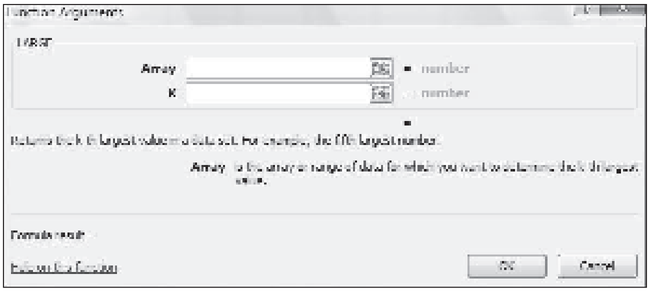


Figure 6-5: The Function Arguments dialog box for LARGE.

PERCENTILE and PERCENTRANK

Closely related to rank is the *percentile*, which represents a score's standing in the group as the percent of scores below it. If you've taken standardized tests like the SAT, you've encountered percentiles. An SAT score in the 80th percentile is higher than 80 percent of the other SAT scores.

Excel's PERCENTILE function enables you to find the value at any percentile. Figure 6-6 shows the Function Arguments dialog box PERCENTILE. The dialog box shows the 75th percentile (the value that's greater than 75 percent of the scores) for the numbers in cells C2 through C16. In this example, the 75th percentile is 72.5.

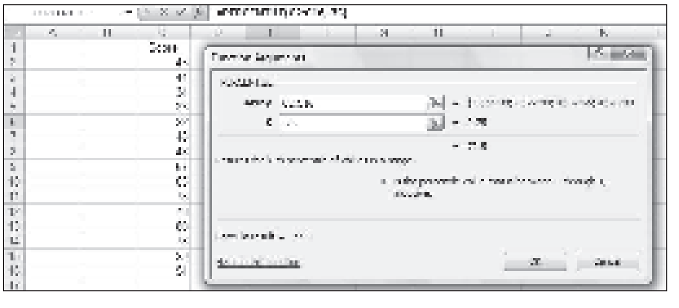


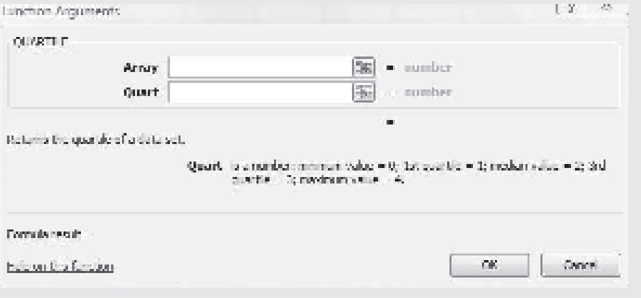
Figure 6-6: The Function Arguments dialog box for PERCENT-ILE.

REMEMBER

In the PERCENTILE dialog box, you enter the percentile into the K box. Enter it as a decimal, so that the 75th percentile is .75.

Drawn and quartiled

A few specific percentiles are often used to summarize a group of scores. The median—the 50th percentile (because it's higher than 50 percent of the scores) — is one of them. Three others are the 25th percentile, the 75th and the 100th percentile (the maximum score). Because they divide a group of scores into fourths, these particular four percentiles are called *quartiles*. Excel's QUARTILE function calculates them. Selecting QUARTILE from the Insert Function dialog box opens the QUARTILE dialog box shown in the figure.



The trick is to enter the right kind of numbers into the Quart box — 1 for the 25th percentile, 2 for the 50th, 3 for the 75th, and 4 for the 100th. Entering 0 into the Quart box gives you the lowest score in the group.

In contrast to percentiles, you might be interested in the flip side: Given a value, what percent of scores falls below it? PERCENTRANK handles this. In Figure 6-7 the Function Arguments dialog box for PERCENTRANK shows the percent rank of 65 for the scores in cells C2 through C16. (It's 0.642, or 64.2 percent.) The Array box holds the array of cells and the X box holds the score (65). The Significance box is optional: You can enter the amount of significant figures in which you would like the answer to appear, or you can leave it blank.

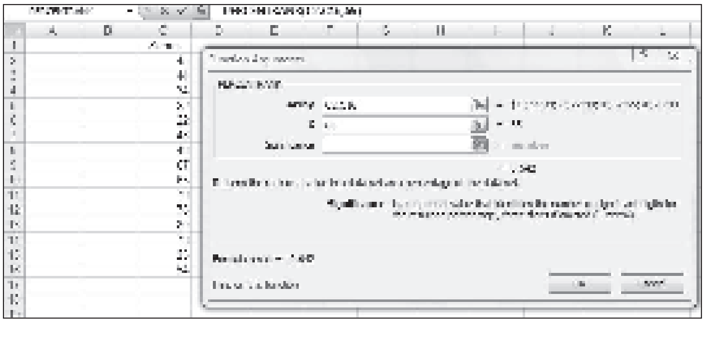


Figure 6-7: The Function Arguments dialog box for PERCENT-RANK.

For the X box, you can enter either the value of a score or the label of the cell in which it appears. In this example, C10 in the X box gives you the same result as putting 65 in the X box.

Data analysis tool: Rank and Percentile

As the name of this section indicates, Excel provides a data analysis tool that calculates ranks and percentiles of each score in a group. The Rank and Percentiles tool calculates both at the same time, so it saves you some steps versus using the separate worksheet functions. (See Chapter 2 to install Excel's data analysis tools.) In Figure 6-8, I take the exam scores from the z-score example and open the Rank and Percentile dialog box.

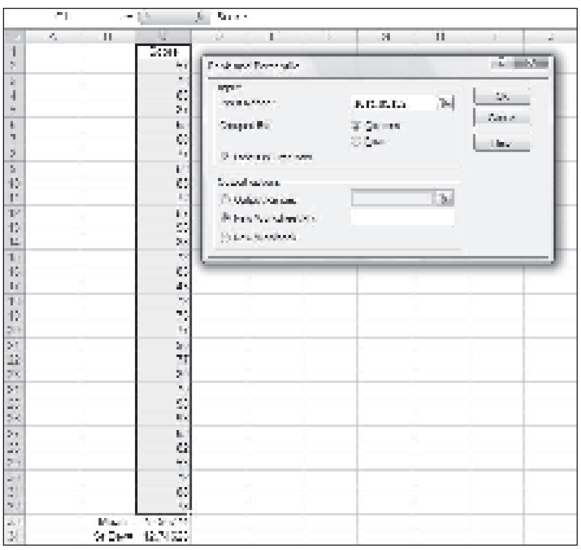


Figure 6-8: The Rank and Percentile analysis tool.

Here are the steps for using Rank and Percentile:

1. **Type your data into an array.**

   In this example, the data are in cells C2 through C32.

2. **In the Tools menu, choose Data Analysis to open the Data Analysis dialog box.**
3. **In the Data Analysis dialog box, select Rank and Percentile.**
4. **Click OK to open the Rank and Percentile dialog box.**

5. **In the Rank and Percentile dialog box, enter the data array into the Input Range box. Make sure that it's in absolute reference format.**

   In this example, a label is in the first row (in cell C1). I want the label included in the output, so I enter $C$1:$C$32 in the Input Range box, and I check the Labels in First Row checkbox.

6. **Click the Columns radio button to indicate that the data are organized by columns.**

7. **Click the New Ply radio button to create a new tabbed page in the worksheet, and to send the results to the newly created page.**

8. **Click OK to close the dialog box. Open the newly created page to see the results.**

Figure 6-9 shows the new page with the results. The table orders the scores from highest to lowest, as the Score column shows along with the Rank column. The Point column tells you the score's position in the original grouping. For example, the 98 in cell B2 is the 12th score in the original data. The Percent column gives the percentile for each score.

	A	B	C	D
1	Point	Score	Rank	Percent
2	12	98	1	96.60%
3	24	98	1	96.60%
4	25	93	3	93.10%
5	20	90	4	90.00%
6	13	88	5	83.30%
7	30	88	5	83.30%
8	4	87	7	80.00%
9	3	85	8	73.30%
10	9	85	8	73.30%
11	15	83	10	66.60%
12	26	83	10	66.60%
13	27	82	12	63.30%
14	22	80	13	60.00%
15	2	78	14	53.30%
16	31	78	14	53.30%
17	7	77	16	43.30%
18	19	77	16	43.30%
19	21	77	16	43.30%
20	18	75	19	40.00%
21	10	72	20	26.60%
22	14	72	20	26.60%
23	17	72	20	26.60%
24	29	72	20	26.60%
25	23	70	24	23.30%
26	5	65	25	20.00%
27	6	60	26	16.60%
28	8	58	27	13.30%
29	1	57	28	6.60%
30	11	57	28	6.60%
31	28	53	30	3.30%
32	16	45	31	0.00%

Figure 6-9: The Output of the Rank and Percentile analysis tool.